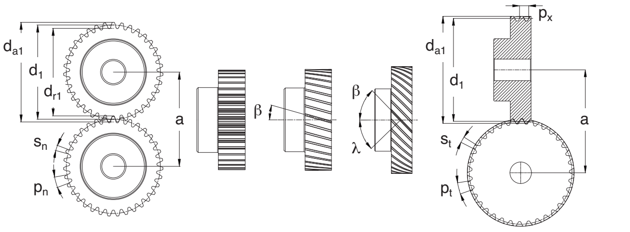
Spur and helical gears

Description	Symbol	Unit	Equation for spur gears	Equation for helical gears
Normal module	m_n			
Transverse module	m_t		$= m_n$	$= m_n / \cos\beta$
Axial module	m_x		-	$= m_n / \sin\beta$
Normal Pressure Angle	α_n	degrees	20°	20°
Transverse Pressure Angle	α_t	degrees	$= \alpha_n$	$= \tan^{-1} (\tan\alpha_n / \cos\beta)$
Helix angle	β	degrees	0°	15° ou 45°
Lead angle	λ	degrees	-	$90-\beta$
Number of teeth	Z			
Profile shift coefficient	x		0 as standard	0 as standard
Addendum	h_a	mm	$1.m_n$	$1.m_n$
Dedendum	h_f	mm	$1.25m_n$	$1.25m_n$
Tooth depth	h	mm	$2.25m_n$	$2.25m_n$
Gear ratio	R		$= Z_2 / Z_1$	$= Z_2 / Z_1$
Centre distance	a	mm	$= (d_1+d_2) / 2$	$= (d_1+d_2) / 2$
Pitch circle diameter	d	mm	$= Z.m_n$	$= Z.m_n=(Z.m_n) / \cos\beta$
Tip diameter	d_a	mm	$= d + (2m_n.x) + (2m_n)$	$= d + (2m_n.x) + (2m_n)$
Root diameter	d_r	mm	$= d_a - (2.h)$	$= d_a - (2.h)$
Normal pitch	p_n	mm	$= \pi.m_n$	$= \pi.m_n$
Transverse pitch	p_t		-	$= \pi.m_t =(\pi.m_n) / \cos\beta$
Axial pitch	p_x		-	$= \pi.m_x =(\pi.m_n) / \sin\beta$
Normal tooth thickness in pitch circle	s_n	mm	$= (p_n/2) + 2m_n.x.\tan\alpha_t$	$= (p_n/2) + 2m_n.x.\tan\alpha_n$
Transversal tooth thickness in pitch circle	s_t	mm	-	$= (p_t/2) + 2m_n.x.\tan\alpha_t$

When working with a pair of gears the subscripts 1 & 2 denote the input (drive) and the output (driven) gear.
Tip diameter is the theoretical diameter of the gear without tooth thickness tolerance applied.
For s_n & s_t when à x=0, this is the theoretical tooth thickness. Actual tooth thickness will be less.
The subscript e is for upper allowance values and i for lower allowance values.
A 15° right handed helical gear must be used with a 15° left handed helical gear.
A 45° right handed helical gear must be used with a 45° left handed helical gear.

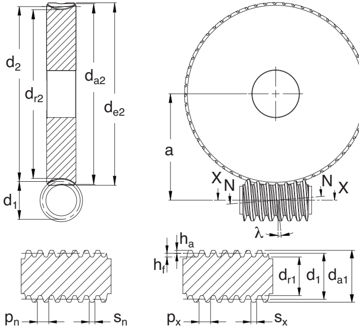
Precision parallel helical gears have a helix angle of 15° and are not compatible with the standard range SH which has a helix angle of 17°45'.



Worms and wheels

Description	Symbol	Unit	Formula
Axial module	m_x		
Normal module	m_n		$= m_x . \sin \lambda$
Normal Pressure Angle	α_n	degrees	$= \tan^{-1} (\tan \alpha_n / \cos \lambda)$
Transverse Pressure Angle	α_t	degrees	20°
Lead angle	λ	degrees	$= \tan^{-1} ((m_x . Z_1) / d_2)$
Helix angle	β	degrees	$90 - \lambda$
Number of starts on worm	Z_1		
Number of starts on wheel	Z_2		
Profile shift coefficient	x		0 as standard
Addendum	h_a	mm	$1.m_x$
Dedendum	h_f	mm	$1.25m_x$
Tooth depth	h	mm	$2.25m_x$
Gear ratio	R		$= Z_2 / Z_1$
Centre distance	a	mm	$= (d_1+d_2) / 2$
Reference diameter of worm	d_1	mm	$(m_x.Z_1) / \tan \lambda$
Reference diameter of wheel	d_2	mm	$= Z_2.m_x$
Tip diameter of worm	d_{a1}	mm	$= d_1 + (2m_x)$
Root diameter of worm	d_{r1}	mm	$= d_{a1} - (2.h)$
Tip diameter of worm	d_{a2}	mm	$= d_2 + (2m_x)$
Root diameter of wheel	d_{r2}	mm	$= d_{a2} - (2.h)$
Outside diameter of wheel	d_{r2}	mm	$= d_{a2} + m_x$
Normal pitch	p_n	mm	$= \pi.m_n$
Axial pitch	p_x	mm	$= \pi.m_x$
Normal tooth thickness in pitch circle	s_n	mm	$= s_x.\cos \lambda$
Transversal tooth thickness in pitch circle	s_t	mm	$= (p_x/2) + 2m_x.x.\tan \alpha_t$

Tip diameter is the theoretical diameter of the gear without tooth thickness tolerance applied.
For s_n & s_t, when x = zero, this is the theoretical tooth thickness. Actual tooth thickness will be less.



Efficiency

Efficiency

The following formulae allows an approximate value for the efficiency of a worm/wheel pair to calculated. The efficiency is dependent on the type of lubrication used (these figures are based on use of mineral oil) and do not take into account bearing, seal and other losses.

$\eta = \tan \lambda / \tan (\lambda + pz)$

$pz = \arctan (\mu)$

$v_g = (d_1 \, . \, n_1) / (19098 \, . \tan \lambda)$

$T_1 = (T_2 / u) * \eta$

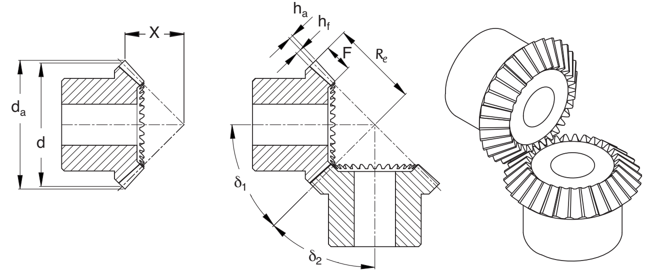
T_1 = Input torque (Nm)
T_2 = Output torque (Nm)
R = Ratio
η = Efficiency
λ = Lead angle (degrees)
μ = Coefficient of friction
pz = Angle of friction
v_g = Sliding speed (m/s)
n_1 = Rotational speed of worm (rpm)
d_1 = Pitch diameter of worm (mm)

Coefficient of friction (Mineral oil)

Speed (m/s)	μ for speeds 0-30m/s									
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0-0.9	0.1500	0.0803	0.0694	0.0623	0.0583	0.0543	0.0521	0.0500	0.0480	0.0459
1.0-1.9	0.0438	0.0423	0.0410	0.0396	0.0382	0.0369	0.0359	0.0352	0.0344	0.0336
2.0-2.9	0.0329	0.0322	0.0316	0.0309	0.0304	0.0297	0.0293	0.0289	0.0286	0.0280
3.0-3.9	0.0276	0.0272	0.0268	0.0265	0.0261	0.0257	0.0254	0.0251	0.0248	0.0245
4.0-4.9	0.0242	0.0239	0.0236	0.0234	0.0232	0.0229	0.0226	0.0224	0.0223	0.0221
5.0-5.9	0.0219	0.0217	0.0215	0.0214	0.0212	0.0210	0.0209	0.0207	0.0205	0.0203
6.0-6.9	0.0202	0.0200	0.0199	0.0197	0.0196	0.0194	0.0193	0.0192	0.0190	0.0189
7.0-7.9	0.0187	0.0186	0.0185	0.0184	0.0183	0.0182	0.0181	0.0179	0.0178	0.0177
8.0-8.9	0.0176	0.0175	0.0174	0.0173	0.0173	0.0172	0.0172	0.0170	0.0169	0.0169
9.0-9.9	0.0169	0.0168	0.0166	0.0166	0.0164	0.0164	0.0164	0.0163	0.0162	0.0162
10.0-10.9	0.0161	0.0160	0.0159	0.0159	0.0159	0.0158	0.0157	0.0156	0.0156	0.0156
11.0-11.9	0.0155	0.0154	0.0154	0.0153	0.0153	0.0152	0.0151	0.0151	0.0150	0.0150
12.0-12.9	0.0149	0.0149	0.0149	0.0148	0.0148	0.0147	0.0147	0.0147	0.0146	0.0146
13.0-13.9	0.0146	0.0146	0.0146	0.0145	0.0145	0.0144	0.0144	0.0144	0.0144	0.0144
14.0-14.9	0.0143	0.0143	0.0143	0.0142	0.0142	0.0142	0.0142	0.0142	0.0141	0.0141
15.0-15.9	0.0141	0.0141	0.0141	0.0140	0.0140	0.0139	0.0139	0.0139	0.0139	0.0139
16.0-16.9	0.0139	0.0138	0.0138	0.0138	0.0138	0.0138	0.0137	0.0137	0.0137	0.0137
17.0-17.9	0.0137	0.0136	0.0136	0.0136	0.0136	0.0136	0.0135	0.0135	0.0135	0.0135
18.0-18.9	0.0135	0.0134	0.0134	0.0134	0.0134	0.0134	0.0134	0.0134	0.0134	0.0134
19.0-19.9	0.0134	0.0133	0.0133	0.0133	0.0133	0.0133	0.0132	0.0132	0.0132	0.0132
20.0-20.9	0.0132	0.0131	0.0131	0.0131	0.0131	0.0131	0.0131	0.0131	0.0131	0.0131
21.0-21.9	0.0131	0.0130	0.0130	0.0130	0.0130	0.0130	0.0130	0.0130	0.0130	0.0130
22.0-22.9	0.0130	0.0129	0.0129	0.0129	0.0129	0.0129	0.0129	0.0129	0.0129	0.0129
23.0-23.9	0.0129	0.0129	0.0128	0.0128	0.0128	0.0128	0.0128	0.0128	0.0128	0.0128
24.0-24.9	0.0128	0.0128	0.0127	0.0127	0.0127	0.0127	0.0127	0.0127	0.0127	0.0127
25.0-25.9	0.0127	0.0127	0.0126	0.0126	0.0126	0.0126	0.0126	0.0126	0.0126	0.0126
26.0-26.9	0.0126	0.0126	0.0125	0.0125	0.0125	0.0125	0.0125	0.0125	0.0125	0.0125
27.0-27.9	0.0125	0.0125	0.0124	0.0124	0.0124	0.0124	0.0124	0.0124	0.0124	0.0124
28.0-28.9	0.0124	0.0124	0.0124	0.0124	0.0124	0.0124	0.0124	0.0124	0.0123	0.0123
29.0-29.9	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123
30.0	0.0123	-	-	-	-	-	-	-	-	-

Bevel gears

Description	Symbol	Unit	Formula
Normal module	m_n		
Pressure angle	α	degrees	20°
Shaft angle	$\sum$	degrees	$= tan^{-1}((m_x . Z_1) / d_2)$
Gear ratio	R		$= Z_2 / Z_1$
Pitch diameter	d	mm	$= Z.m_n$
Pitch cone angle	δ_1	degrees	$= tan^{-1}(\sin\sum / (R+\cos\sum))$
Pitch cone angle	δ_2	degrees	$= \sum - \delta 1$
Cone Distance	R_e	mm	$= d_2 / 2\sin\delta_2$
Addendum	h_a	mm	$1.m_n$
Dedendum	h_f	mm	$1.25.m_n$ (mod 0.6 to 1)
			$1.22.m_n$ (mod 1.5 to 2)
			$1.20.m_n$ (mod 4)
Outside diameter	d_a	mm	$= d + 2h_a.\cos\delta$
Pitch apex to crown	X	mm	$= R_e.\cos\delta - h_a.\sin\delta$



Backlash

The backlash figures given for spur, helical and crossed axis helical gears is the theoretical backlash for two identical gears at standard centre distance to the ISO 286 centre distance tolerance.
It is given as circumferential backlash in mm measured on pitch circle diameter. An upper and lower value is quoted.
Theoretical backlash is the difference between tooth thickness without and with tolerance applied.
Backlash is calculated according to DIN 3967

Tooth thickness tolerance

Gear type	Module 0.5 to 0.8	Module 1.0 to 3.0	Centre distance tolerance
Spur	7e/8e DIN 58405	e25 DIN 3967	Js7
Spur (Skive hobbed)	6e DIN 58405	e25 DIN 3967	Js7
Pinion	7e DIN 58405	e25 DIN 3967	-
Parallel helical	7e DIN 58405	e25 DIN 3967	Js7
Crossed axis helical	7e DIN 58405	e25 DIN 3967	Js8
Worm + wheel	7e/8e DIN 58405	e25 DIN 3967	Js8

Gear type	Module 0.6 to 4.0
Bevel	7f24 DIN 3965/3967

A_{sn} : *Tooth thickness allowance which is the difference between measured gear tooth thickness and theoretical value measured in the normal section. When working with a pair of gears the subscripts 1 & 2 denote the input (drive) and the output (driven) gear.*
For worm and wheel, 1 relates to the worm and 2 to the wheel.
The subscript e is for upper allowance and i for lower allowance.

T_{sn} : *Tooth thickness tolerance measured in the normal section. (mm)*

$$A_{sne} = S_n - S_{ne}$$

$$A_{sni} = A_{sne} - T_{sn} = S_n - S_{ni}$$

Circumferential backlash Jt

This is the length of arc on the pitch circle diameter through which each can be rotated whilst the other is held stationary. It is measured in the transverse section. Units are mm and degrees

$$j_t = \frac{A_{sn1} + A_{sn2}}{\cos\beta} + \Delta j_a$$

Normal Backlash Jn

This is the shortest distance between the flanks of the gears when the opposite flanks are in contact. It is measured in the transverse section. Units are mm and degrees

For spur, helical, crossed axis helical gears:

$$j_n = j_t \bullet \cos\alpha_n \bullet \cos\beta$$

Backlash

Change in circumferential backlash due to Centre Distance Tolerance ΔJ_a

Units =mm and degrees

$$\Delta j_a = 2 \bullet A_s \bullet \frac{\tan \alpha_n}{\cos \beta}$$

Spur Gear		Parallel Helical Gear		Crossed Axis Helical Gear	
Deviation from centre distance A_s	Change in backlash ΔJ_a	Deviation from centre distance A_s	Change in backlash ΔJ_a	Deviation from centre distance A_s	Change in backlash ΔJ_a
0,001	0,001	0,001	0,001	0,001	0,001
0,010	0,007	0,010	0,008	0,010	0,010
0,015	0,011	0,015	0,011	0,015	0,015
0,020	0,015	0,020	0,015	0,020	0,021
0,025	0,018	0,025	0,019	0,025	0,026
0,030	0,022	0,030	0,023	0,030	0,031
0,035	0,025	0,035	0,026	0,035	0,036
0,040	0,029	0,040	0,030	0,040	0,041
0,045	0,033	0,045	0,034	0,045	0,046
0,050	0,036	0,050	0,038	0,050	0,051

Angular Backlash

Units =mm and degrees

$$j_\theta = \frac{360 \text{ x } j_t}{\pi \text{ x } d_2}$$

d_2 = *Reference diameter (mm)*

A_s = *Centre distance tolerance (i.e. a = 30mm Js7, As = ±0.0105mm)*

α_n = *Normal pressure angle (α_n = 20°)*

β = *Helix angle (β = zero for spur gears)*

Replace helix angle β with lead angle λ for worm and wheel.

1° = 60 *arc minutes*

e25 DIN 3967

Reference diameter From	To	Upper tooth thickness allowance A_{sne}	Tooth thickness allowance T_{sn}
-	10	-0.022mm	0.020mm
10	50	-0.030mm	0.030mm
50	125	-0.040mm	0.040mm
125	280	-0.056mm	0.050mm

e25 DIN 58405

Reference diameter d (mm)	Normal module m_n	Upper tooth thickness allowance A_{sne}	Tooth thickness allowance T_{sn}
From 3 to 6	Since 0.16 to 0.25	0.028	0.011
	Since 0.25 to 0.6	0.030	0.012
	Since 0.6 to 1.6	0.035	0.014
> 6 to 12	Since 0.16 to 0.25	0.030	0.012
	Since 0.25 to 0.6	0.035	0.014
	Since 0.6 to 1.6	0.040	0.016
> 12 to 25	Since 0.16 to 0.25	0.035	0.014
	Since 0.25 to 0.6	0.040	0.016
	Since 0.6 to 1.6	0.045	0.018
	Since 1.6 to 3	0.050	0.020
> 25 to 50	Since 0.16 to 0.25	0.040	0.016
	Since 0.25 to 0.6	0.045	0.018
	Since 0.6 to 1.6	0.050	0.020
	Since 1.6 to 3	0.055	0.022
> 50 to 100	Since 0.16 to 0.25	0.045	0.012
	Since 0.25 to 0.6	0.050	0.018
	Since 0.6 to 1.6	0.055	0.020
	Since 1.6 to 3	0.063	0.022
> 100 to 200	Since 0.6 to 1.6	0.063	0.024
	Since 1.6 to 3	0.070	0.029
> 200 to 400	Since 0.6 to 1.6	0.070	0.029
	Since 1.6 to 3	0.080	0.032

Backlash

Example of calculating the backlash for two non-identical gears

Input (Driving) gear: PSG0.5-20 7e
Ouput (Driven) gear: PSG0.5-40 7e

1• *Calculate the reference diameter d for each gear*
PSG0.5-20 $d_1 = z * m_n = 10.00mm$
PSG0.5-40 $d_2 = 20.00mm$

2• *Find A_{sne} and T_{sn} from the tables on previous page*
PSG0.5-20 $A_{sne} = -0.035mm$ $T_{sn} = -0.014mm$
PSG0.5-40 $A_{sne} = -0.040mm$ $T_{sn} = -0.016mm$

3• *Calculate A_{sni} for each gear*
PSG0.5-20 $A_{sni} = A_{sne} - T_{sn} = -0.035-0.014 = -0.021mm$
PSG0.5-40 $A_{sni} = A_{sne} - T_{sn} = -0.040-0.016 = -0.024mm$

4• *Calculate the centre distance of the two gears and the centre distance tolerance*
centre distance = (10 + 20) / 2 = 15*mm*
Js7 = 0,009*mm*

5• *Calculate the change in backlash due to centre distance*

$$j_a = 2 \bullet A_s \bullet \frac{\tan \alpha_n}{\cos \beta} + 2 \bullet 0.009 \bullet \frac{\tan 20}{\cos 0} = 0.007mm$$

6• *Calculate the maximum backlash (Remove the minus sign on A_{sn})*

$$j_t = \frac{A_{sni1} + A_{sni2}}{\cos \beta} + \Delta j_a = \frac{0.035 + 0.040}{\cos 0} + 0.007 = 0.082mm$$

7• *Calculate the minimum backlash (Remove the minus sign on A_{sn})*

$$j_t = \frac{A_{sne1} + A_{sne2}}{\cos \beta} + \Delta j_a = \frac{0.021 + 0.024}{\cos 0} - 0.007 = 0.038mm$$

8• *Convert to angular backlash*

$$j_\theta = \frac{360 \times j_t}{\pi \times d_2}$$ 1° = 60 *arc minutes*

$j_\theta =$ 28.208 to 13,072 *arc minutes*